

Cyclic Universe and Infinite Past

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Abstract

We address two questions about the past for infinitely cyclic cosmology. The first is whether it can contain an infinite length null geodesic into the past in view of the Borde-Guth-Vilenkin (BGV) "no-go" theorem, The second is whether, given that a small fraction of spawned universes fail to cycle, there is an adequate probability for a successful universe after an infinite time. We give positive answers to both questions then show that in infinite cyclicity the total number of universes has been infinite for an arbitrarily long time.

In a recent article [1] a model for an infinitely cyclic universe was proposed which addresses the entropy problem [2] by appealing to phantom dark energy and a deflation mechanism which reduces the large entropy accrued during expansion to zero [3] which is the constant adiabatic value during contraction.

In this way, the entropy at turnaround is all displaced to a volume exterior to our universe and this enables a cyclic scenario to be consistent provided that the dark energy has an equation of state satisfying $w = p/\rho < -1$. It does not matter how much below -1 it is, although observationally (and even anthropically [3]) it is easier to distinguish from a cosmological constant the bigger is $\phi = |w + 1|$.

Several legitimate concerns must be addressed before we accept that this is the correct solution about entropy adopted by Nature. A couple of them will be addressed in this Letter. But first let us enumerate a few concerns that will not be addressed here but which one hopes to discuss in the future.

One issue is the accomplishment of a turnaround as described in [1] where there is an abrupt disintegration into a large number $1/f^3$ of causal patches, each of which separately contracts into a new universe. The scale factor changes from $a(t)$ for expansion to $\hat{a}(t) = fa(t)$ for contraction. No less important is a better understanding of the bounce. Both of these issues involve the critical density term for which existing derivations are based on extra dimensions [4,5]. A technical treatment in four spacetime dimensions is desirable in each case to reassure us that the model is consistent without commitment to string theory or extra dimensions. We know of no fatal flaw.

In the present article, our more modest aim is to address the question of the infinite past during which necessarily there were an infinite number of turnarounds and bounces each uniformly spaced by the constant time period τ .

Theorists are comfortable with an infinite future as occurs in the standard model with a cosmological constant. In that case the universe expands exponentially forever, and other galaxies recede from ours to become invisible. Entropy gradually increases.

There seems to be less widespread acceptance of an infinite past. One reason is the old worry about entropy [2] that it must increase and so at a finite time in the past would fall to zero. This is avoided in [1]. Another possible concern is provided by arguments about null geodesics into the past and whether the spacetime manifold can be past complete; this is alleviated in the present Letter.

It is true that the infinite past is less familiar than the infinite future, and surely an infinite past requires a cyclic model. At each turnaround a small fraction (f) of universes will fail to cycle and we confirm that^{#1} there is a high likelihood that after infinite cycles we live in a successful universe. There is no reason that an infinite past is less viable than an infinite future, although as we shall show it does require somewhat unfamiliar concepts such as the inevitability of infinite cyclicity and that the total number of universes has been infinite for an arbitrarily long time.

^{#1}It was D. Reichart who raised and answered this question of success versus failure.

Past null geodesics

There is a general argument about past completeness of the spacetime manifold which we address first.

We begin with the no-go theorem of [6] which we shall adapt for application to the more general case in [1], as the original no-go theorem applies to past inflation. We shall show how this no-go theorem is by-passed, as the assumptions no longer apply.

The metric is of the form

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \quad (1)$$

In this metric for a null geodesic the affine parameter λ follows the relation

$$d\lambda \propto a(t)dt \quad (2)$$

We normalize the affine parameter to the present time $t = t_0$ by choosing with $a_0 = a(t_0)$

$$d\lambda = \left[\frac{a(t)}{a_0} \right] dt \quad (3)$$

so that $d\lambda/dt = 1$ when $t = t_0$.

Following [6], We multiply Eq.(2) by the Hubble parameter $H = \dot{a}/a$ where a dot denotes derivative with respect to t but now we integrate from an initial time $t_n = t_0 - n\tau$ up to $t = t_0$ to obtain with $a_n = a(t_0 - n\tau)$, $\lambda_n = \lambda(t_0 - n\tau)$

$$\int_{\lambda_n}^{\lambda_0} H(\lambda) d\lambda = \frac{1}{a_0} \int_{a_n}^{a_0} da = nC \quad (4)$$

where in the cyclic model we have denoted the finite integral

$$\frac{1}{na_0} \int_{a_n}^{a_0} da = C \quad (5)$$

by the constant C .

The left hand side of Eq.(4) can be written as the average of the Hubble parameter

$$H_{av} \equiv \frac{1}{(\lambda_0 - \lambda_n)} \int_{\lambda_n}^{\lambda_0} H(\lambda) d\lambda \quad (6)$$

over n cycles. In particular, it is important that H_{av} in Eq.(6) is independent of the integer n because of cyclicity.

Given Eq.(6), we find from Eq.(4) that

$$H_{av} = \lim_{n \rightarrow \infty} \left[\frac{nC}{(\lambda_0 - \lambda_n)} \right] \quad (7)$$

so that for $n \rightarrow \infty$, we find a backwards null geodesic $(\lambda_0 - \lambda_n) \propto n$ of infinite length and the argument of [6] does not apply. Such a geodesic is exemplified by a photon propagating always at the origin $\mathbf{x} = 0$ of the spatial coordinates.

Whether or not the past incompleteness arguments apply to the competing cyclic model of [7–9], we take no position. In [6], it is argued that they do, but the authors disagree [10], so that jury is still out. But we do assert that they do not apply to the model in [1], as can be seen directly from our Eq. (7), where $(\lambda_0 - \lambda_n)$ necessarily becomes an infinite length past null geodesic for $n \rightarrow \infty$, given the finiteness of both H_{av} and C .

Successful and failed universes

Now we turn to another issue. At each turnaround, a very large number N of new universes is spawned. Let the number of universes at time $t = t_0 - n\tau$ be Σ_n . Then the total number now is $\Sigma_0 = N^n \Sigma_n$.

This is not quite right because although almost every causal patch contains no photons and no matter, a tiny fraction $f \ll 1$ will contain one or more photons and hence because of pair production will fail to cycle and bounce prematurely. Similarly any other matter such as a quark or lepton in the causal patch will cause failure. According to [3], this number is very small, generally $f < 1/N$ but we need to examine the failed universes to assess the probability that we may live in a successful rather than a failed universe now, after an infinite $n \rightarrow \infty$ of cycles.

Let us ignore any new universes spawned by failed universes. The number of successful universes is given after n cycles by

$$\begin{aligned} \Sigma_0^{(successful)} &= \lim_{n \rightarrow \infty} [\Sigma_n (N - fN)^n] \\ &= \lim_{n \rightarrow \infty} \Sigma_n [(1 - f)N]^n \end{aligned} \quad (8)$$

The number of failed universes, on the other hand, is

$$\begin{aligned} \Sigma_0^{(failed)} &= \lim_{n \rightarrow \infty} \Sigma_n [fN + fN(1 - f)fN + fN[(1 - f)N]^2 + \dots + fN[(1 - f)N]^{(n-1)}] \\ &= \lim_{n \rightarrow \infty} \Sigma_n fN [(1 - f)N]^{(n-1)} [1 - \{1/(1 - f)N\}]^{-1} \\ &= \lim_{n \rightarrow \infty} \Sigma_n fN [(1 - f)N]^n [(1 - f)N - 1] \end{aligned} \quad (9)$$

The probability for a successful universe at present is given by the ratio of Eq.(8) with the sum of Eq.(8) and Eq.(9) which gives

$$P^{(successful)} = \frac{[(1 - f)N - 1]}{(N - 1)} \quad (10)$$

which for $N \gg 1$ and $f \ll 1$ is approximately $P^{successful} = (1 - f)$ similarly to each single turnaround, as expected.

This is non-trivial when both subsets are infinite and if it had been that failed universes dominate instead, the model would have been untenable because our universe would be infinitely unlikely. Fortunately, this is not the case.

Total number of universes

Last but certainly not least, we study the total number of universes versus time in the past.

Suppose that $\Sigma_n < \infty$ for some finite n . Then going back another n' cycles we have $\Sigma_{n+n'} = \Sigma_n N^{-n'}$. N satisfies $N > 1$ (actually $N \gg 1$) so for some n' the integral part of $\Sigma_{n+n'} = 1$ and cyclicity fails. Therefore no finite Σ_n is permitted for any finite n . In particular, the present number of universes must be $\Sigma_0 = \infty$, as expected after an infinite number of cycles.

More subtle is the value of

$$\begin{aligned}\Sigma_\infty &= \lim_{n \rightarrow \infty} (\Sigma_0 N^{-n}) \\ &= \Sigma_0 [\lim_{n \rightarrow \infty} (N^{-n})]\end{aligned}\tag{11}$$

which is indeterminate as the product of infinity (Σ_0) times zero. This requires some recourse to cardinality and the transfinite numbers of set theory [11,12], depending on the level of rigor demanded.

In set theory the lowest transfinite is \aleph_0 (Aleph-zero) and the simplest assumption is that the number of universes is always \aleph_0 , the cardinality of the primes, the integers or the rational numbers. When \aleph_0 is multiplied by a finite number N , it remains \aleph_0 . This holds for any finite n in Eq.(11) and so extends back an arbitrarily long time in the past. For the infinite past, one cannot really say anything from Eq.(11).

At each turaround the number of universes increase by a gigantic factor N but $N \times \infty = \infty$ so in that sense the total number remains infinite.

The process is not time-reversal invariant and the global entropy of all universes increases with time consistent with the second law of thermodynamics. Considering only our universe, however, the entropy as well as the density and temperature are cyclic and never infinite. This is as near to infinite cyclicity as seems possible consistent with statistical laws. The old problem confronting Tolman [2] is avoided by removing entropy to an unobservable exterior region; one may say in hindsight that the problem lay in considering only one universe.

Summary

We have argued that an infinite past time is consistent and that the cyclic model of [1] is an exemplar. The presence of patches which fail to cycle is not a problem as after an infinite number of cycles the probability of being in a successful universe as we find ourselves is practically one. Also, it is mandatory that the total number of universes is infinite for an arbitrarily long time into the past. We assume the total number of universes is constant and equal to \aleph_0 (Aleph-zero). This idea is unfamiliar but appears to us to be an inevitable concomitant of an infinite past.

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